

# A proof of the Navier interfacial equation through quantitative comparison between solid-polymer melt and solid-elastomer friction

Marceau Hénot,<sup>1</sup> Éric Drockenmuller,<sup>2</sup> Liliane Léger,<sup>1</sup> and Frédéric Restagno<sup>1,\*</sup>

<sup>1</sup>*Laboratoire de Physique des Solides, CNRS, Univ. Paris-Sud,  
Université Paris-Saclay, 91405 Orsay Cedex, France*

<sup>2</sup>*Univ Lyon, Université Lyon 1, CNRS, Ingénierie des Matériaux Polymères, UMR 5223, F-69003, Lyon, France*  
(Dated: April 13, 2017)

The slip behavior of polydimethylsiloxane (PDMS) polymer melts flowing on non-adsorbing surfaces made of short non-entangled PDMS chains densely end-grafted to silica has been characterized. For high enough shear rates, constant slip lengths proportional to the bulk fluid viscosity have been observed, in agreement with Navier's interfacial equation, and demonstrating that the interfacial Navier's friction coefficient is a local quantity, independent of the polymer molecular weight. Comparing the interfacial shear stresses deduced from these measured slip lengths to available friction stress measured for crosslinked PDMS elastomers, we directly compared the interfacial melt or elastomer friction coefficient to the monomer-monomer friction.

If the Navier-Stokes equation is often considered as one of the most difficult one, the choice of a boundary condition to describe a liquid flowing is often considered as trivial. The relative tangential fluid velocity is assumed to be zero. Historically, Navier introduced a more general condition [1]: the shear stress at the solid-liquid interface should be proportional to the component of the fluid velocity tangent to the surface  $V$ :

$$\sigma_{\text{fluid} \rightarrow \text{surface}} = kV \quad (1)$$

where  $k$  is an interfacial solid-liquid friction coefficient, assumed to be independent of the shear rate. As for a Newtonian fluid, the shear stress at the interface is proportional to the shear rate, the interfacial friction coefficient is usually converted into the so called Navier's slip length  $b = \eta/k$ . It represents the distance from the surface at which the velocity profile extrapolates to zero. The determination of slip lengths for simple fluids has been the subject of intensive experimental [2–7] and theoretical [8–11] researches over the last 20 years. The majority of experiments or numerical simulations can be summarized as follow: i) the linear relation in Eq. 1 can be used for simple liquids at not too high shear rates, ii) the interfacial friction coefficient  $k$  depends on the solid-liquid interactions, iii) expected or measured slip lengths lie between 0 to 50 nm. We lack however of quantitative prediction based on starting principles for the slip length of simple liquids. This is because the measured values are always small and highly sensitive to tiny molecular details of the surfaces, in particular nanometric scales geometrical and chemical heterogeneities [3, 5].

Contrary to simple liquids, polymer melts can present slip lengths larger than the size of the molecules. Polymer melts thus are interesting candidates to quantitatively test Eq. 1 and possibly get rid of the molecular details of the surface. Indirect experimental evidences for a giant slip of polymers melts were provided by extrusion instabilities reported since the 40's [12], extensively

studied by polymer rheologists [13]. In 1979, de Gennes proposed a simple explanation of such a huge slip, for polymer melts [14] flowing on ideal non-adsorbing surfaces. The physical idea is that  $k$ , which results from the local contact between monomers and the solid wall, should be independent of chain entanglements and chain length, while entanglements do control the polymer melt viscosity. One should thus simply observe a slip length proportional to the polymer viscosity:

$$b(N) = \frac{\eta(N)}{k} \quad (2)$$

where  $N$  the degree of polymerization of the chains. De Gennes proposed an estimate of  $k$  by assuming that the slip length for a fluid of monomers  $b(1)$  should be of the order of the monomer size:

$$b(1) = \eta_0/k \approx a \quad (3)$$

Similar argument based on kinetic theory, have also lead to predict an increase of the slip length with the polymer viscosity [10, 11] for non entangled polymer melts. Few attempts to test quantitatively Eq. 2 have been reported despite the potential interest in terms of testing the Navier's hypothesis. An important experimental issue is the strong tendency of polymer molecules to adsorb at surfaces. Adsorption of few polymer molecules on the surface has been reported to be responsible for a strongly non linear slip behavior, characterized by three different slip regimes depending on the shear rate: for low shear rates, entanglements between surface adsorbed and bulk chains produce a large friction and thus a weak slip, while for high enough shear rates, bulk and surface chains become stretched and fully disentangled, leading to high slip. In between, the slip length has been observed to increase linearly with the shear rate, instead of Eq. 1 [15–21]. The large shear rate regime has been identified by some authors for polymer melts, but not fully characterized, due to the difficulty of spanning a large enough shear rate range above the critical shear rate, in order to be sure that the slip lengths are indeed shear rate independent. Durliat *et al.* observed such a

\* Corresponding author : frederic.restagno@u-psud.fr

shear independent high slip regime, for one PDMS molecular weight flowing on weakly dense PDMS grafted layers ([22]); Mhetar and Archer [16] investigated the molecular weight dependence and measured slip lengths scaling like  $b \sim M^{1.3}$  on weakly adsorbing surfaces due to some defects on their surfaces. Wang and Drda [23] reported a slip length at the onset of high slip scaling like  $b \sim M^{3.5}$ . More recently, Dewetting experiments [24, 25], reported slip lengths proportional to the viscosity for high molecular weights melts. All these studies indicate that high slip can indeed be obtained with polymer melts at large enough shear rates and tend to show that the slip lengths do scale with bulk viscosities. They however lack two crucial facets needed to understand the phenomenon at the molecular level: a clear investigation of the shear rate dependence, and an independent determination of the interfacial friction coefficient,  $k$ , so that the validity of the Navier equation could be fully tested.

We present here direct proofs of both Navier and de Gennes hypothesis, based first on direct measurements of the slip lengths for PDMS melts with various molecular weights and narrow molecular weight distributions, flowing on silica surfaces decorated with densely packed end-anchored non-entangled short PDMS chains, and second, on direct friction measurements of crosslinked PDMS elastomers sliding on the same surface.

All polymer fluids used were trimethylsiloxy terminated PDMS melts with weight average molecular weight  $M_w$ , obtained by controlled fractionation of a commercial batch (Petrarch PS047.5), and mixed with 0.5% by weight of fluorescently labeled photobleachable PDMS chains with a weight average molecular weight  $M_w^* = 321 \text{ kg}\cdot\text{mol}^{-1}$  and polydispersity index 1.18. The fluorescent chains were lab-synthesized and labeled at both ends with nitrobenzoxadiazole groups (NBD) (emission wavelength at 550 nm [26, 27] when excited at 458 nm. The molecular parameters of the polymers melts are shown in Table I. The surfaces on which slip was investigated were the polished surface of a fused silica prism, covered with end grafted short PDMS chains with an average molecular weight  $2 \times 10^3 \text{ g}\cdot\text{mol}^{-1}$ , well below the average molecular weight between entanglements,  $M_e \approx 10 \times 10^3 \text{ g}\cdot\text{mol}^{-1}$  for PDMS [28, 29].

$M_w \text{ (kg}\cdot\text{mol}^{-1})$	609	787	962
polydispersity index	1.15	1.22	1.27
$\eta \text{ (kPa}\cdot\text{s)}$	19.8	43	78
$\Sigma \text{ (chains}\cdot\text{nm}^{-2})$	0.9	0.9	0.6

TABLE I. Molecular characteristics of the PDMS melts and of the PDMS grafted surfaces.  $M_w$  is the weight average molecular weight,  $\eta$  is the bulk zero shear viscosity of the melts, obtained by extrapolating the data from [26].  $\Sigma$  is the grafting density of the surface layer made of  $2 \text{ kg}\cdot\text{mol}^{-1}$  end grafted PDMS chains. The grafting density is expressed in number of chains per  $\text{nm}^2$ .

The synthesis protocol of these end functionalized chains along with the grafting procedure are detailed in

Léger *et al.* [29]. The grafting densities are reported in Table I, the grafted chains are almost in the stretched regime.

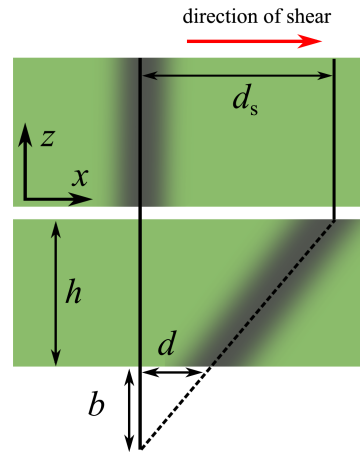


FIG. 1. Principle of velocimetry using photobleaching. Top: schematics of the photobleached line inside in the fluid. Bottom: the liquid has been sheared over a distance  $d_s$ , with a no slip boundary condition at the top plate while it has slipped at the bottom plate over a distance  $d$  which corresponds to a slip length  $b$ .

The experimental technique used to measure the slip lengths is described in details in the supplementary material. It is an improved version of the velocimetry technique described by Léger *et al.* [15]. We briefly recall here its principle. As shown in Figure 1, a line is photobleached all through the fluid before applying any shear. The deformations of that line after the application of a simple shear for a chosen time, are analyzed in order to extract the eventual slip distances at both upper and lower surfaces and the true average shear rate experienced by the fluid. The improvements we have brought essentially consist in: using imaging and imaging analysis techniques to produce a two dimension analysis of the initial and final bleached lines, with spatial resolution in the plane parallel to the surface. These spatially resolved information are then used to reconstruct the displacement profile inside the fluid containing the velocity and the normal to the surface, with a micrometric resolution in the direction normal to the surface. A typical image showing both an initial and final photobleached lines is reported in Figure 2. For all investigated samples, we observed no slip at the top plate, which had been cleaned in order to favour adsorption, while all fluids were observed to slip at the prism surface over a distance  $d$ . The slip lengths measured for the three investigated melts are reported in Figure 3 as a function of the shear rate experienced by the fluid for shear rates between  $0.1$  and  $4 \text{ s}^{-1}$ . For the two lower molecular weights, two slip regimes are observed: at low shear rates,  $b$  depends on  $\dot{\gamma}$  while above a critical shear rate  $\dot{\gamma}^*$  which depends on the sample, a slip regime with a constant slip length appears. Such a transition has yet been reported in the literature by Massey *et al.* [17] and has been attributed

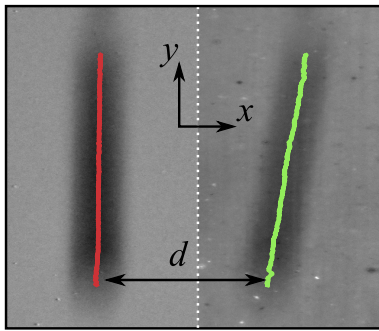


FIG. 2. View from above of the photobleached line. Left: initial fluorescence image, with the bleached line. Right: image obtained after shear (deformed bleached line on the right). For each  $y$  value, the fluorescence intensities are analyzed, and the central position determined (respectively in red and green). The set of distances between the two lines gives access to the displacement profile inside the fluid provided the angle between the laser beam and the  $y$  axis,  $\theta$  is known (see supplementary material for detail).

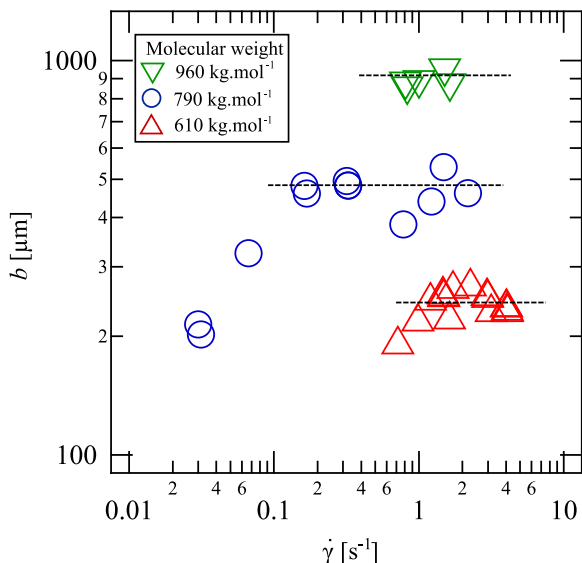


FIG. 3. Slip lengths as a function of the shear rate experienced by the fluid for melts of molecular weight 609, 787 and 962  $\text{kg}\cdot\text{mol}^{-1}$ . The horizontal lines represent the average slip lengths in the constant slip regime for each molecular weight.

to few adsorbed long chains on the surface [21]. Due to the large polydispersity of loops and tails in case of adsorbed chains, the transition appears progressive. However, for large enough shear rates, the surface attached chains are fully disentangled from the melt chains. Quite recently, Chennevière *et al.* based on neutrons reflectivity experiments, could probe directly such an expulsion of the melt chains from the surface layer for high shear rates [30]. In the high shear rate regime, we observe shear rate independent slip lengths, meaning that the Navier's hypothesis of a linear response at the interface, and thus of a friction coefficient independent of the shear rate, is indeed correct for these systems. It is also well visible

in Figure 3 that the slip lengths strongly depend on the molecular weight of the melt. To go a step further in the analysis of the validity of the Navier's hypothesis, these average measured slip lengths are compared to the zero shear bulk viscosities of the various melts used in the present study in Figure 4. The values of the zero

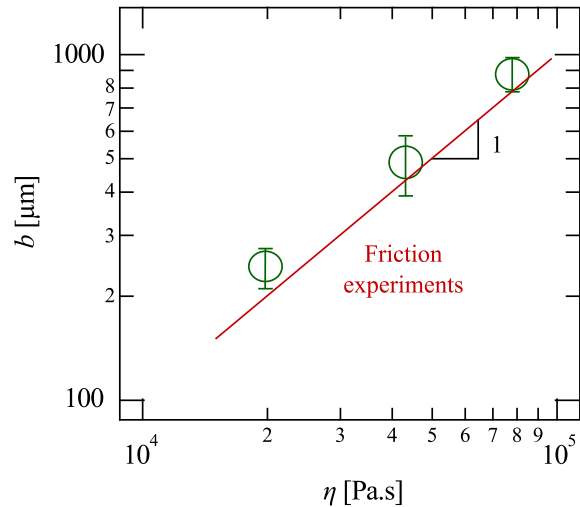


FIG. 4. Slip length as a function of the viscosity of the melts. The red line of slope 1 is the prediction of de Gennes's model with no adjustable parameters using the friction coefficient obtained from friction experiments [31].

shear viscosities of the melts could not be measured directly, due to the very small volume of liquid available. They have been extrapolated from the data in [26]. The clear linear relationship between the slip length and the viscosity provides a direct proof of the full validity of the Navier's hypothesis for PDMS melts in contact with dense grafted PDMS Brushes of short chains. This linear relationship not only means that the Navier's hypothesis of a linear response is correct, but it also demonstrates that the friction coefficient  $k$  is indeed independent of the chain length as predicted by de Gennes in 1979 [14]. A linear fit of these data provides a value for the interfacial friction coefficient, with no need of any other hypothesis:  $k_{\text{slippage}} = 8.5 \times 10^7 \text{ kg}\cdot\text{m}^{-1}\cdot\text{s}^{-1}$ . Using the estimate proposed by de Gennes [14] (Eq. 3) with our measured  $k$  value and a microscopic viscosity  $\eta(1) \sim 0.001 \text{ Pa}\cdot\text{s}$  for the fluid of monomers leads to  $b_0 \sim 10^{-11} \text{ m}$ . This is much smaller than a molecular size  $a$ . Bäumchen *et al.* [18] obtained quite similar results from dewetting experiments. Comparing their friction measurements to the de Gennes model, these authors proposed that, in order to account for the small friction coefficient, the number of entanglements close to the surface should be smaller than in the bulk. We prefer to keep the idea that  $k$  is a local quantity, which cannot give information on large scale organization of the polymer chains, as reflected by the fact that it does not depend on the molecular weight of the chains. In fact, the estimate of  $b(1)$  proposed by de Gennes in Eq. 3 is not based on strong argument, and slip lengths much smaller than the molecular size have been

reported for simple liquids, (see [6] for a review) both from experiments and from molecular dynamics simulations.

To further test both the Navier's and de Gennes' hypothesis it then appears crucial to look for an independent measurement of the interfacial friction coefficient  $k$ . The slip lengths measured in the present study

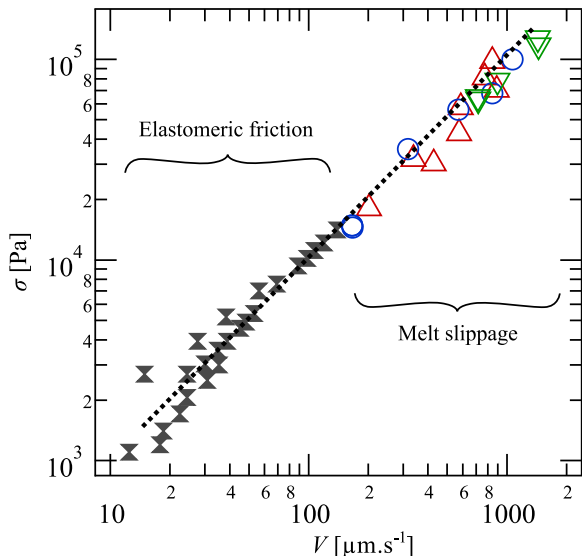


FIG. 5. Tangential stress exerted by PDMS melts or crosslinked elastomers on a grafted layer of short PDMS chains. Empty symbols represent data for the melts used in this study, calculated using data of Figure 3a and viscosities given in Table I. The legend of these markers are the same than in Figure 3. Black symbols are data corresponding to a crosslinked PDMS lens, reproduced from Cohen *et al.* [31].

have thus been compared to direct friction experiments between cross-linked PDMS elastomers and the same grafted PDMS surface, as suggested by Ghatak *et al.* [32]. Bureau [33] and Cohen [31] investigated the friction of small cross-linked PDMS lenses on the same grafted layers that used in this study. For soft elastomer-solid contacts, the friction is not characterized by a solid friction coefficient since the apparent and real contact areas are equal. Instead, the significant physical quantity is the tangential stress  $\sigma$  exerted by the elastomer sphere on the solid surface. This stress was measured at different sliding velocities  $V$  by Bureau *et al.* and Cohen *et al.* who obtained a linear relationship between  $\sigma$  and  $V$ ,  $\sigma(V) = \sigma_0 + k_{\text{friction}}V$ , with  $\sigma_0$  depending on the quality on the grafted layers. The experimental data  $\sigma(V) - \sigma_0$

from Cohen *et al.* [31] are reproduced in Figure 5 (black markers). Using equations 1 and 2, the interfacial stress exerted by the three PDMS melts on the surfaces in our slip measurements were calculated, and are shown in Figure 5 (empty colored markers). The friction coefficient extracted from the direct crosslinked elastomer/surface friction experiments corresponds in Figure 5 to the dashed line. This value of the friction coefficient fits perfectly the slip data, even when extrapolated upward by an order of magnitude. The polymer melt slip experiments and the crosslinked elastomer friction experiments do give the same friction coefficient. At the molecular level, this friction should be a monomer-monomer friction, as for both systems everything is made out of PDMS as already pointed by Bureau *et al.* [33]. A monomer-monomer friction coefficient can be deduced from rheological bulk properties of PDMS melts. The data of Barlow *et al.* [34] on silicon oils, give a monomer-monomer friction coefficient  $\zeta_0 = 8.6 \times 10^{-9} \text{ dyn}\cdot\text{s}\cdot\text{cm}^{-1}$ . Self diffusion experiments [26], also give the exact same value for  $\zeta_0$ . The monomer-monomer friction coefficient and the interfacial friction coefficient  $k$  are related through the scaling  $k = \zeta_0/a_{\text{rheology}}^2$ . Comparing the  $k$  value measured in our experiments to the monomer-monomer friction coefficient measured independently, the prefactor in equation can be extracted. This gives  $a_{\text{rheology}} \approx 1 \text{ nm}$ , indeed of the order of magnitude of what is expected for a molecular size.

To conclude, using a velocimetry technique based on photobleaching, we measured the slip length for three PDMS melts of different viscosity flowing on a weakly-adsorbing surfaces. We observed shear rate independent slip lengths for high enough shear rates proportional to the fluid viscosity as predicted by Navier. This linear dependence of the high shear rate slip length versus the fluid viscosity also provided a strong test of the de Gennes's model [14] for polymer melts flowing on ideal surfaces. We showed that the friction coefficient of the melts flowing on grafted layer of short PDMS chains is equal to the one previously measured by direct solid friction measurements of a crosslinked PDMS elastomers on the same surfaces [31, 33]. This allowed to directly connect for the first time solid-solid friction to solid-fluid friction in a direct measurement. Finally, we showed that a molecular prediction of the solid-fluid or the solid-liquid friction coefficient is possible by using the monomer-monomer friction that can be extracted from a rheology experiment.

This work was supported by ANR-ENCORE program (ANR-15-CE06-005) and European Research Council Grant (FP7/2007-2013). We thank F. Boulogne, A. Chennevière for their technical help.

[1] H. Navier, Mem. Acad. Sci. Inst. Fr **6**, 389 (1823).  
[2] D. Y. C. Chan and R. G. Horn, The Journal of Chemical Physics **83**, 5311 (1985).  
[3] R. Pit, H. Hervet, and L. Léger, Physical Review Letters **85**, 980 (2000).

[4] C. Cottin-Bizonne, B. Cross, A. Steinberger, and E. Charlaix, Phys. Rev. Lett. **94**, (2005).  
[5] T. Schmatko, H. Hervet, and L. Léger, Physical Review Letters **94**, 244501 (2005).  
[6] C. Neto, D. R. Evans, E. Bonaccorso, H.-J. Butt, and

- V. S. J. Craig, Reports on Progress in Physics **68**, 2859 (2005).
- [7] E. Lauga, M. Brenner, and H. Stone, in *Springer handbook of experimental fluid mechanics* (Springer) pp. 1219–1240.
- [8] L. Bocquet and J.-L. Barrat, Physical Review E **49**, 3079 (1994).
- [9] P. Thompson and S. Troian, Nature **389**, 360 (1997).
- [10] N. V. Priezjev and S. M. Troian, Physical Review Letters **92** (2004), 10.1103/PhysRevLett.92.018302.
- [11] L. Bocquet and J.-L. Barrat, Soft Matter **3**, 685 (2007).
- [12] C. J. Petrie and M. M. Denn, AIChE Journal **22**, 209 (1976).
- [13] N. El Kissi and J. Piau, Journal of Non-Newtonian Fluid Mechanics **37**, 55 (1990).
- [14] P. G. De Gennes, C. R. Acad. Sc. Paris (1979).
- [15] L. Léger, H. Hervet, G. Massey, and E. Durliat, Journal of Physics: Condensed Matter **9**, 7719 (1997).
- [16] V. Mhetar and L. A. Archer, Macromolecules **31**, 8607 (1998).
- [17] G. Massey, H. Hervet, and L. Léger, Europhysics Letters (EPL) **43**, 83 (1998).
- [18] R. Fetzer, M. Rauscher, A. Münch, B. Wagner, and K. Jacobs, EPL (Europhysics Letters) **75**, 638 (2006).
- [19] F. Brochard and P. G. De Gennes, Langmuir **8**, 3033.
- [20] K. B. Migler, H. Hervet, and L. Léger, Phys. Rev. Lett. **70**, 287 (1993).
- [21] F. Brochard-Wyart, C. Gay, and P. G. de Gennes, Macromolecules **29**, 377 (1996).
- [22] E. Durliat, H. Hervet, and L. Léger, Europhysics Letters **38**, 383 (1997).
- [23] S.-Q. Wang and P. P. Drda, Macromol.Chem.Phys. **198**, 673 (1997).
- [24] O. Bäumchen, R. Fetzer, and K. Jacobs, Physical Review Letters **103** (2009), 10.1103/PhysRevLett.103.247801.
- [25] J. D. McGraw, O. Bäumchen, M. Klos, S. Haefner, M. Lessel, S. Backes, and K. Jacobs, Advances in Colloid and Interface Science **210**, 13.
- [26] L. Léger, H. Hervet, P. Auroy, E. Boucher, and G. Massey, in *Rheology for Polymer Melt Processing*, Rheology Series, Vol. 5, edited by J.-M. Piau and J.-F. Agassant (Elsevier, 1996) pp. 1–16.
- [27] C. Cohen, D. Damiron, S. B. Dkhil, E. Drockenmuller, F. Restagno, and L. Léger, Journal of Polymer Science Part A: Polymer Chemistry **50**, 1827 (2012).
- [28] L. J. Fetters, D. J. Lohse, D. Richter, T. A. Witten, and A. Zirkel, Macromolecules **27**, 4639 (1994).
- [29] L. Léger, E. Raphaël, and H. Hervet, “Surface-anchored polymer chains: Their role in adhesion and friction,” in *Polymers in Confined Environments* (Springer Berlin Heidelberg, Berlin, Heidelberg, 1999) pp. 185–225.
- [30] A. Chennevière, F. Cousin, F. Boué, E. Drockenmuller, K. R. Shull, L. Léger, and F. Restagno, Macromolecules **49**, 2348 (2016).
- [31] C. Cohen, F. Restagno, C. Poulard, and L. Léger, Soft Matter **7**, 8535.
- [32] A. Ghatak, K. Vorvolakos, H. She, D. L. Malotky, and M. K. Chaudhury, The Journal of Physical Chemistry B **104**, 4018 (2000).
- [33] L. Bureau and L. Léger, Langmuir **20**, 4523.
- [34] A. J. Barlow, G. Harrison, and J. Lamb, Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences **282**, 228 (1964).
- 
- [1] H. Navier, Mem. Acad. Sci. Inst. Fr **6**, 389 (1823).
- [2] D. Y. C. Chan and R. G. Horn, The Journal of Chemical Physics **83**, 5311 (1985).
- [3] R. Pit, H. Hervet, and L. Léger, Physical Review Letters **85**, 980 (2000).
- [4] C. Cottin-Bizonne, B. Cross, A. Steinberger, and E. Charlaix, Phys. Rev. Lett. **94**, (2005).
- [5] T. Schmatko, H. Hervet, and L. Léger, Physical Review Letters **94**, 244501 (2005).
- [6] C. Neto, D. R. Evans, E. Bonaccorso, H.-J. Butt, and V. S. J. Craig, Reports on Progress in Physics **68**, 2859 (2005).
- [7] E. Lauga, M. Brenner, and H. Stone, in *Springer handbook of experimental fluid mechanics* (Springer) pp. 1219–1240.
- [8] L. Bocquet and J.-L. Barrat, Physical Review E **49**, 3079 (1994).
- [9] P. Thompson and S. Troian, Nature **389**, 360 (1997).
- [10] N. V. Priezjev and S. M. Troian, Physical Review Letters **92** (2004), 10.1103/PhysRevLett.92.018302.
- [11] L. Bocquet and J.-L. Barrat, Soft Matter **3**, 685 (2007).
- [12] C. J. Petrie and M. M. Denn, AIChE Journal **22**, 209 (1976).
- [13] N. El Kissi and J. Piau, Journal of Non-Newtonian Fluid Mechanics **37**, 55 (1990).
- [14] P. G. De Gennes, C. R. Acad. Sc. Paris (1979).
- [15] L. Léger, H. Hervet, G. Massey, and E. Durliat, Journal of Physics: Condensed Matter **9**, 7719 (1997).
- [16] V. Mhetar and L. A. Archer, Macromolecules **31**, 8607 (1998).
- [17] G. Massey, H. Hervet, and L. Léger, Europhysics Letters (EPL) **43**, 83 (1998).
- [18] R. Fetzer, M. Rauscher, A. Münch, B. Wagner, and K. Jacobs, EPL (Europhysics Letters) **75**, 638 (2006).
- [19] F. Brochard and P. G. De Gennes, Langmuir **8**, 3033.
- [20] K. B. Migler, H. Hervet, and L. Léger, Phys. Rev. Lett. **70**, 287 (1993).
- [21] F. Brochard-Wyart, C. Gay, and P. G. de Gennes, Macromolecules **29**, 377 (1996).
- [22] E. Durliat, H. Hervet, and L. Léger, Europhysics Letters **38**, 383 (1997).
- [23] S.-Q. Wang and P. P. Drda, Macromol.Chem.Phys. **198**, 673 (1997).
- [24] O. Bäumchen, R. Fetzer, and K. Jacobs, Physical Review Letters **103** (2009), 10.1103/PhysRevLett.103.247801.
- [25] J. D. McGraw, O. Bäumchen, M. Klos, S. Haefner, M. Lessel, S. Backes, and K. Jacobs, Advances in Colloid and Interface Science **210**, 13.
- [26] L. Léger, H. Hervet, P. Auroy, E. Boucher, and G. Massey, in *Rheology for Polymer Melt Processing*, Rheology Series, Vol. 5, edited by J.-M. Piau and J.-F. Agassant (Elsevier, 1996) pp. 1–16.
- [27] C. Cohen, D. Damiron, S. B. Dkhil, E. Drockenmuller, F. Restagno, and L. Léger, Journal of Polymer Science Part A: Polymer Chemistry **50**, 1827 (2012).
- [28] L. J. Fetters, D. J. Lohse, D. Richter, T. A. Witten, and A. Zirkel, Macromolecules **27**, 4639 (1994).
- [29] L. Léger, E. Raphaël, and H. Hervet, “Surface-anchored

- polymer chains: Their role in adhesion and friction,” in *Polymers in Confined Environments* (Springer Berlin Heidelberg, Berlin, Heidelberg, 1999) pp. 185–225.
- [30] A. Chennevière, F. Cousin, F. Boué, E. Drockenmuller, K. R. Shull, L. Léger, and F. Restagno, *Macromolecules* **49**, 2348 (2016).
  - [31] C. Cohen, F. Restagno, C. Poulard, and L. Léger, *Soft Matter* **7**, 8535.
  - [32] A. Ghatak, K. Vorvolakos, H. She, D. L. Malotky, and M. K. Chaudhury, *The Journal of Physical Chemistry B* **104**, 4018 (2000).
  - [33] L. Bureau and L. Léger, *Langmuir* **20**, 4523.
  - [34] A. J. Barlow, G. Harrison, and J. Lamb, *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* **282**, 228 (1964).